These calculations are prepared by means of CalcTEX November 9, 2008
http://sg.bzip.pl/CalcTeX and based on W.L. McCabe & J.C. Smith "Unit Operations of Chemical Engineering" 3rd edition.
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Flow calculation

Water with a density of $\rho_w := 998.0 \cdot [\text{kg}] / [\text{m}]^3$ enters a $D_a := 50.0 \cdot [\text{mm}]$ pipe fitting horizontally, as show in Fig. 1, at a steady velocity of $V_a := 1.0 \cdot [\text{m}] / [\text{s}]$ and a gauge preassure of $p_a := 100.0 \cdot [\text{kPa}]$. It leaves the fitting horizontally, at the same elevation, at an angle of $\vartheta := \pi/4$ with the entrance direction. The diameter at the outlet is $D_b := 20.0 \cdot [\text{mm}]$. Assuming the fluid density is constant, that the kinetic-energy and momentum correction factors at both entrance and exit are unity, and that the friction loss in the fitting is negligible, calculate:

(a) the guague pressure at the exit of the fitting and

(b) the force in the x and y directions exerted by the fitting on the fluid.

Solution

(a)
$V_a \cdot [\text{m}] / [\text{s}] = 1.0$, $V_a \cdot [\text{ft}] / [\text{s}] = 3.28083989501$, $D_a \cdot [\text{inch}] = 1.96850393701$, $D_b \cdot [\text{inch}] = 0.787401574803$, $\rho_w \cdot [\text{lb}] / [\text{ft}]^3 = 62.303104655$.
From Eq. (1)

$$\frac{\rho_a}{\rho_b} \cdot \frac{V_a}{V_b} = \left(\frac{D_b}{D_a}\right)^2 \tag{1}$$

$$V_b := V_a \cdot \left(\frac{D_a}{D_b}\right)^2; \quad V_b \cdot [\text{m}] / [\text{s}] = 6.25; \quad V_b \cdot [\text{ft}] / [\text{s}] = 20.5052493438$$

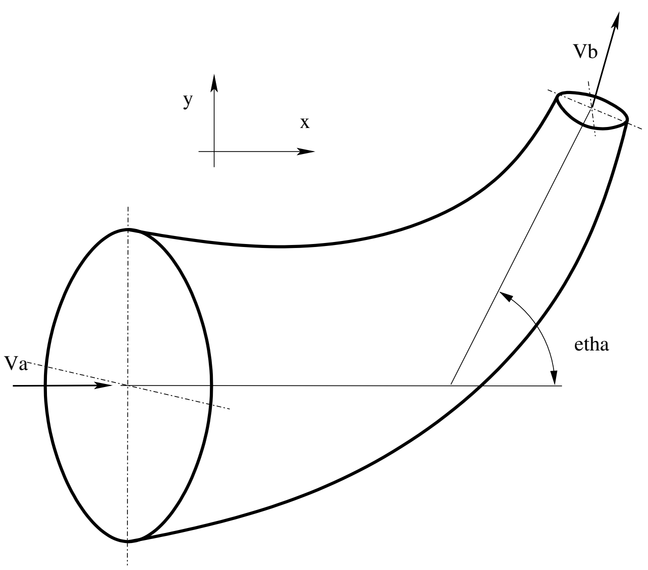

Figure 1: Flow through reducting fitting, viewed from the top

The outlet preassure p_b is found from Eq. (2).

$$\frac{p_a}{\rho} + \frac{g \cdot Z_a}{g_c} + \frac{\alpha_a \cdot V_a^2}{2 \cdot g_c} = \frac{p_b}{\rho} + \frac{g \cdot Z_b}{g_c} + \frac{\alpha_b \cdot V_b^2}{2 \cdot g_c} + h_f \tag{2}$$

Since $Z_a = Z_b$ and h_f my be neglected, Eq. (2) becomes

$$\frac{p_a - p_b}{\rho} = \frac{V_b^2 - V_a^2}{2}$$

from which

$$p_b := p_a - \rho_w \cdot \frac{V_b^2 - V_a^2}{2.0}; \;\; p_b \cdot [\text{kPa}] = 81.0068125$$

Note the $g_c := 1.0$ – is omitted when working in SI units.

(b)

The forces acting on the fluid are found by combining Eqs. (3) and (4)

$$\sum F = \frac{\dot{m}}{g_c} \cdot (\beta_b \cdot V_b - \beta_a \cdot V_a) \tag{3}$$

$$\sum F = p_a \cdot S_a - p_b \cdot S_b + F_w - F_g \tag{4}$$

where:

F_w – net force of wall of chanel on fluid,

F_g – component of force of gravity (written for flow in upward direction). For the x direction, since $F_g := 0$ for horizontal flow, this gives

$$\dot{m}(\beta_b V_{b_x} - \beta_a V_{a_x}) = p_a S_{a_x} - p_b S_{b_x} + F_{w_x} \tag{5}$$

where S_{a_x} and S_{b_x} are the projected areas of S_a and S_b on planes normal to the flow direction. (Recall that preassure p is a scalar quantity.) Since the flow enters in the x direction, $V_{a_x} = V_a$ and

$$S_a := \pi \cdot D_a^2/4.0; \quad S_a \cdot [\text{cm}]^2 = 19.6349540849; \quad S_{a_x} := S_a$$

From Fig. 1

$$V_{b_x} := V_b \cdot \cos(\vartheta); \quad V_{b_x} \cdot [\text{m}] \,/\, [\text{s}] = 4.41941738242$$

$$S_b := \pi \cdot D_b^2/4.0; \quad S_b \cdot [\text{cm}]^2 = 3.14159265359; \quad S_b \cdot [\text{inch}]^2 = 0.486947835202$$

Also

$$S_{b_x} := S_b \cdot \cos(\vartheta); \quad S_{b_x} \cdot [\text{cm}]^2 = 2.22144146908; \quad S_{b_x} \cdot [\text{inch}]^2 = 0.344324116356$$

From Eq. (6)

$$\dot{m} = \rho_a \cdot V_a \cdot S_a = \rho_b \cdot V_b \cdot S_b = \rho \cdot V \cdot S \tag{6}$$

$$\dot{m} := V_a \cdot \rho_w \cdot S_a; \;\; \dot{m} \cdot [\text{kg}] \,/\, [\text{s}] = 1.95956841768$$

Substituting in Eq. 5 and solving from F_{w_x} gives

$$F_{w_x} := \dot{m} \cdot (V_{b_x} - V_a) - p_a \cdot S_{a_x} + p_b \cdot S_{b_x}; \quad F_{w_x} \cdot [\text{kN}] = -0.171653769283$$

Similarly, for the y direction $V_{a_y} := 0$ and $S_{a_y} := 0$, and

$$V_{b_y} := V_b \cdot \sin(\vartheta); \quad V_{b_y} \cdot [\text{m}] \,/\, [\text{s}] = 4.41941738242; \quad V_{b_y} \cdot [\text{ft}] \,/\, [\text{s}] = 14.4994008609$$

$$S_{b_y} := S_b \cdot \sin(\vartheta); \quad S_{b_y} \cdot [\text{cm}]^2 = 2.22144146908; \quad S_{b_y} \cdot [\text{inch}]^2 = 0.344324116356$$

Hence

$$F_{w_y} := \dot{m} \cdot \left(V_{b_y} - V_{a_y}\right) - p_a \cdot S_{a_y} + p_b \cdot S_{b_y}; \quad F_{w_y} \cdot [\text{N}] = 26.6553399837.$$